

2010 Q10.

(a)

$$y \frac{dy}{dx} = x + xy^2$$

$$\Rightarrow y \frac{dy}{dx} = x(1 + y^2)$$

$$\Rightarrow y dy = x(1 + y^2) dx$$

$$\Rightarrow \frac{y}{1+y^2} dy = x dx$$

$$\Rightarrow \int \frac{y}{1+y^2} dy = \int x dx$$

$$\Rightarrow \frac{1}{2} \ln(1+y^2) = \frac{x^2}{2} + C$$

$$y = 0 \text{ when } x = 0 \Rightarrow C = 0$$

$$\Rightarrow \frac{1}{2} \ln(1+y^2) = \frac{x^2}{2}$$

$$\Rightarrow \ln(1+y^2) = x^2$$

$$\Rightarrow e^{x^2} = 1+y^2$$

$$\Rightarrow \sqrt{e^{x^2} - 1} = y$$

(b)(i)

$$V \frac{dV}{dx} = 0.12 - 0.0006V^2$$

$$\Rightarrow \frac{V}{0.12 - 0.0006V^2} = dx$$

$$\Rightarrow \int_0^V \frac{V}{0.12 - 0.0006V^2} = \int_0^{120} dx$$

$$\Rightarrow \left[-\frac{1}{0.0012} \ln(0.12 - 0.0006V^2) \right]_0^V = \left[x \right]_0^{120}$$

$$\Rightarrow \left(-\frac{1}{0.0012} \cdot \ln(0.12 - 0.0006V^2) \right) - \left(-\frac{\ln(0.12)}{0.0012} \right) = 120 - 0$$

$$\Rightarrow \frac{1}{0.0012} \left(\ln 0.12 - \ln(0.12 - 0.0006V^2) \right) = 120$$

$$\Rightarrow \frac{1}{0.0012} \left(\ln \frac{0.12}{0.12 - 0.0006V^2} \right) = 120$$

$$\Rightarrow \ln \frac{0.12}{0.12 - 0.0006V^2} = 0.144$$

$$\Rightarrow e^{0.144} = \frac{0.12}{0.12 - 0.0006V^2}$$

$$\Rightarrow e^{0.144} (0.12 - 0.0006V^2) = 0.12$$

$$\Rightarrow 0.12 - 0.0006V^2 = \frac{0.12}{e^{0.144}}$$

$$\Rightarrow 0.12 - 0.0006V^2 = 0.1039$$

$$\Rightarrow -0.0006V^2 = -0.01609$$

$$\Rightarrow V^2 = 26.822$$

$$\Rightarrow V = 5.18 \text{ ms}^{-1}$$

$$(ii) \quad S = \frac{D}{T}$$

$$\Rightarrow T = \frac{D}{S}$$

$$\Rightarrow T = \frac{120}{2.65}$$

$$\Rightarrow T = 45.28 \text{ Sec.}$$